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Principles and Techniques of Satellite Navigation Using the Timation II Satellite

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Space Systems Division

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CONTENTS

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This is an interim report; work continues.

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PRINCIPLES AND TECHNIQUES OF SATELLITE NAVIGATION USING THE TIMATION II SATELLITE

INTRODUCTION

The Timation* experiment for satellite navigation is being developed under the sponsorship of the Naval Air Systems Command. The Timation II satellite was launched Sept. 30, 1969, and is still operating at this date. Timation II transmits range and doppler information near 150 and 400 MHz, which can be used to correct range or doppler for first-order ionospheric refraction. Five U.S.-based ground stations are used to track the satellite and collect telemetry information from the sensors on board the satellite. Other ground stations are used to control the satellite's subsystems, including the ability to tune (in phase and frequency) the on board quartz crystal oscillator.

The overall physical configuration for Timation II is given in Fig. 1. Timation II is equipped with a high-precision quartz crystal oscillator capable of frequency stabilities on the order of a few parts in 10^{11} per day. Timation II is equipped with active thermal control of the oscillator environment, which effectively eliminates oscillator frequency fluctuation due to temperature changes.

Ranging information is provided by means of coherent modulation of the carrier, with modulation frequencies varying from 100 Hz to 1 MHz. The range receiver synthesizes a similar set of frequencies which are phase compared with the received signal.

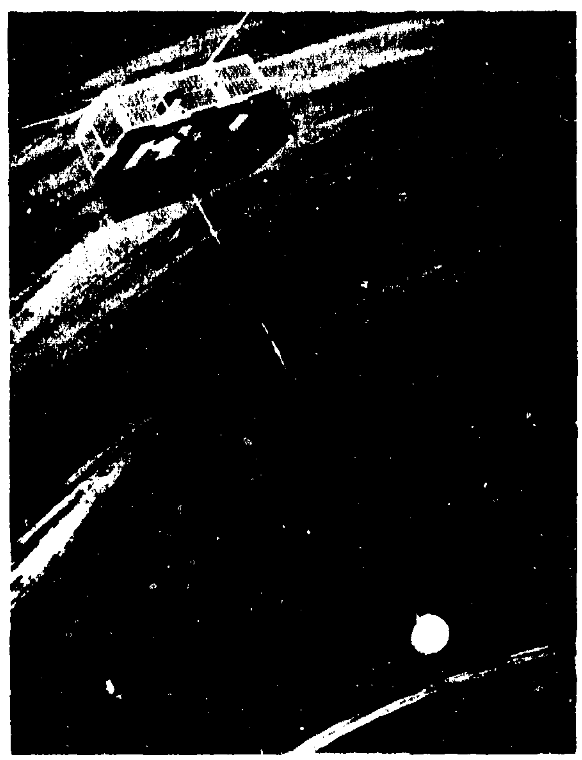

Fig. 1 - The Timation satellite.

The doppler measurements are obtained by adding a phase-locked tracking filter to the Timation range receiver. The filter is used to track the doppler signal; the period of heterodyned doppler is averaged over a specified number of cycles, and the results are printed for subsequent data analysis.

In this report the equations used in the analysis are developed, and the biases of the range and doppler measuring systems are identified with the time difference (range bias) and the frequency difference (doppler bias) between the satellite's and observer's clock or frequency standard. A sensitivity analysis is performed to determine the influence of geometry and noise on navigation accuracy.

*The name Timation is an acronym of Time Navigation.

THE TIMATION II SATELLITE

The Timation II (Fig. 1) satellite has an overall configuration similar to the Timation I (1) satellite; hence only a summary of the features will be given in this report. The satellite weighs approximately 125 lb and consumes an average of 18 W of power furnished by solar cells and batteries. Two-axis gravity gradient stabilization is provided by using an extendable boom. Temperature control is achieved by (a) careful design of the satellite (2) to provide a temperature range from 0°C to +20°C inside the satellite and (b) active temperature control of the quartz crystal frequency standard to maintain its external temperature to within a fraction of a degree. Linearly polarized dipoles are used for the 150- and 400-MHz antennas. A separate telemetry antenna is used. This is mounted on the side and has more than 40 dB of isolation from the main antennas. In addition, a magnetometer is used to sense attitude changes of the satellite.

The frequency of the oscillator may be electromechanically tuned in discrete steps of approximately 3.6×10^{-12}/pulse. In addition, the phase of its transmissions may be advanced or retarded in discrete steps of 33.3 ns/pulse. These two features provide precise control over the satellite clock synchronization and clock rate.

Timation II is in a 500-naut-mi near-circular orbit which has an inclination of 70° to the equatorial plane. With this orbit, several passes of 12 to 16 min duration will be available during the day at each of the five Timation tracking stations.

TIMATION RANGING CONCEPT

The Timation II satellite carries a highly stable crystal oscillator from which nine modulation frequencies of the two carriers of 150 and 400 MHz are obtained. The modulation frequencies are 100 Hz, 312.5 Hz, 1 kHz, 3.125 kHz, 10 kHz, 31.250 kHz, 100 kHz, 312.5 kHz, and 1 MHz. The transmitted modulation frequencies can be received and phase compared with a similar set of coherent tones synthesized from an oscillator or "clock" at the receiver site. This system is thus a frequency interferometer which will measure the "time" difference between the received signal and the local "time" with ambiguities of 80 ms based on the highest common divisor of 12.5 Hz and an accuracy based on the precision of the phase comparison of the highest tone (1 MHz). In the system the resolution of the phase comparison is 1% of a period, giving a time resolution of 10 ns when using the 1-MHz "tone." The accuracy of the "time" comparison of the received and local signals is slightly higher than 10 ns due to phasemeter "zero" adjustment, nonlinearity, differential phase shift in the receiver, noise, etc. This measurement may be converted to ranging information by multiplying by c, the speed of light in a vacuum. This ranging information, which depends on the navigator's position, also includes information on the time difference between the satellite clock and the navigator's clock.

The actual "time" difference between the received signal and the local reference is the "time" difference between the satellite oscillator or "clock" and the ground clock, plus the propagation time required for the signal to propagate from the satellite to the receiver. The "time" indicated by the components of the received signal is subject to some error due to the dispersive effect of the ionosphere.

The user's time base is obtained from the user's frequency standard using suitable countdown and comparison circuitry. (The results of time transfer via Timation II will be discussed in a later report.) The timing requirements for the ground station clock are higher than for the user's clock. The five ground stations are equipped with cesium-beam frequency standards which are kept in time synchronization to the UTC time base.

Time transfer or time comparison between two ground clocks can be made by comparing each to the satellite clock and combining the results. There are two conditions under which this can be done: (a) the case in which locations of the ground clocks are accurately known, and (b) the case with unknown locations. These two cases will be discussed later in the report.

The system user, or navigator, is not required to have the same precision frequency standard as required for use in the satellite. For example, quartz crystal frequency standards with stabilities on the order of a few parts in 10^{10}/day would be suitable for use by a Timation II user. Several American manufacturers have relatively-low-priced quartz crystal frequency standards which meet this requirement.*

A practical question of interest is how a navigator would initially set his clock and proceed to use the navigation satellite. A simple line of reasoning will be used to show how this question can be resolved.

Figure 2 depicts the 500-naut-mi near-circular orbit obtained for Timation II. Reference to Fig. 2 shows that, due to the orbit chosen for Timation II, the range for a navigator on or near the Earth must be between 500 and 1900 naut mi, which corresponds to propagation times varying from 3 to 12 ms. Now for the purposes of discussion, say that the satellite transmits ephemeris information on the minute, which includes the date, hour, and minute of transmission. Two cases will be considered.

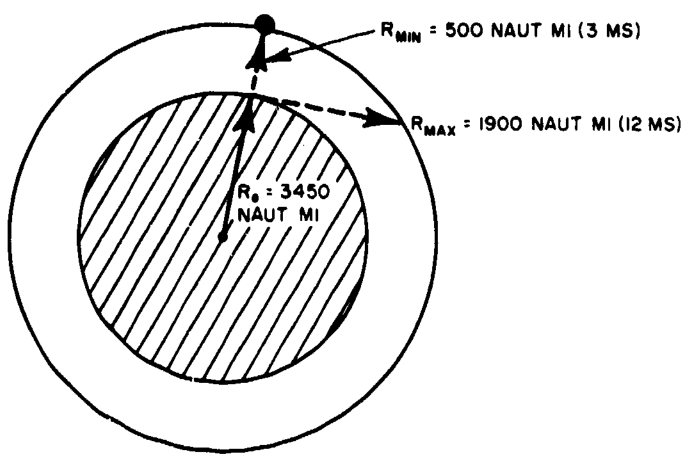

Fig. 2 - Schematic of 500-naut-mi near-circular orbit obtained for Timation II. The minimum and maximum ranges from an observer, on or near the earth, to the satellite is shown.

Case 1 — Navigator's Position on Earth Unknown

Assume that the navigator is receiving information from the satellite. After the satellite transmits the first minute mark, the navigator adjusts his clock to read that minute, plus 7.5 ms (7.5 ms is the average transmission time). His clock will now be synchronized to within ±5 ms of the satellite clock. The navigator now proceeds to take

*See Ref. 3 for a survey on the recent advances in frequency standards.

data until the satellite goes below his horizon. He should now note the *measured* range for the last minute of observable data. That range must be (see Fig. 2) near 12 ms; hence another correction may be made to bring the navigator's clock to within ±1 ms of the satellite clock. Note that so far no knowledge of the navigator's position has been assumed, except that he is on or near the earth.

Case 2 — Position Approximately Known

Since the ranging system is modulo 80 ms and the time is known to ±1 ms, the ranging system may now be used to provide a much higher precision time synchronization. This is done by navigating while using all of the ranging information obtained as the satellite sweeps overhead. However, to do this, an approximate position must be assumed to solve the nonlinear equations involved. The exact radius of convergence required for these equations is not known, except that it is large compared with any reasonable uncertainty in position. For example, convergence has been obtained using an *assumed position* of 1000 naut mi further away from the known position. Needless to say, this should be sufficient to cover most situations of interest.

TIMATION II RANGE AND DOPPLER MEASUREMENT

Figure 3 shows a simplified block diagram of the Timation II 400-MHz range receiver. The Timation II satellite is tracked using a high-gain directional antenna followed by a preamplifier and two stages of IF at frequencies of 370 and 25 MHz. The received tones (varying from 100 Hz to 1 MHz) are then detected, using a double detection technique for the lower tones, and phase compared with a similar set of tones (synthesized from the 5-MHz frequency standard) to produce the phase difference which may be combined to yield the range measurement.

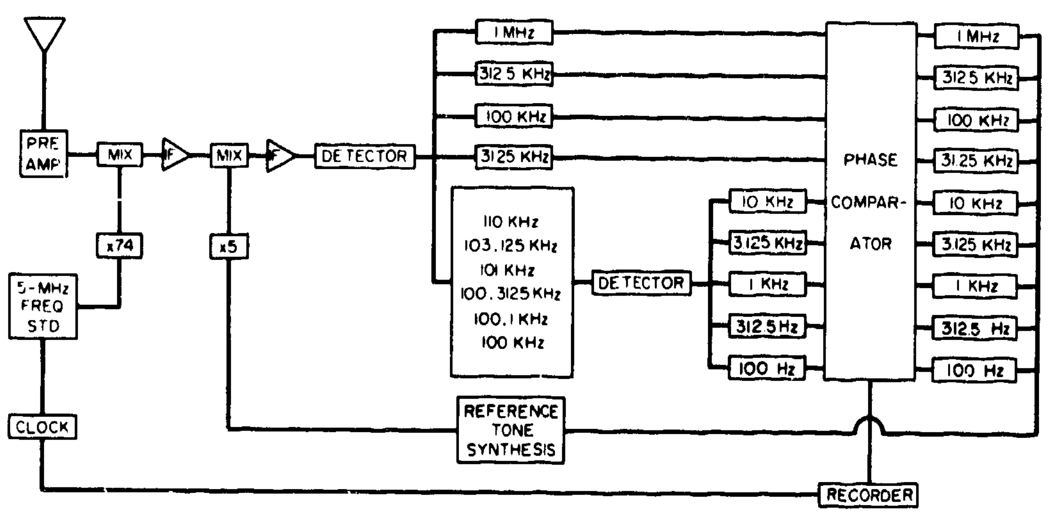

Fig. 3 - Block diagram of the Timation II 400-MHz range receiver.

Doppler measurements are obtained only at the NRL ground station. The NRL ground station is equipped with a phase-locked tracking filter to measure the satellite's doppler shift at 399.4 MHz. This is accomplished by heterodyning a 4.38-MHz signal (synthesized from the 5-MHz standard) with the 4.4-MHz signal available as a result of the two stages of mixing used to derive the ranging information. The procedure results in the doppler signal being offset by a constant 20 kHz The phase-locked loop is then used to track the signal, which is counted and printed for subsequent data reduction.

SATELLITE TRAJECTORY CALCULATIONS

The satellite trajectory computation is made by the Naval Weapons Laboratory (NWL) using doppler tracking data obtained from the NWL tracking network (TRANET). Although the range data from the five Timation ground stations has been included in the orbit determination, it is not routinely included because of the data communications required of the NWL operating system.

The orbit determination is performed on the NWL STRETCH computer using their ASTRO (4) program which performs a statistical estimate of the dynamical and observational parameters of the state variables at epoch. The force model accounts for accelerations from the following sources: (a) earth gravitational accelerations, (b) sun and moon gravitational accelerations, (c) solar and lunar tidal bulge effects, (d) atmospheric drag, and (e) radiation pressure. The earth's gravitational acceleration includes coefficients for the earth's gravitational potential as a function of longitude as well as latitude. Other parameters such as drag and the position of the tracking stations are included in the model. A weighted least-squares estimate is then performed based on observational data obtained over time arcs ranging from 2 to 5 days.

The Timation II satellite is equipped with dual frequencies, which allows for the measurement of first-order ionospheric refraction. With the inclusion of the ionospheric refraction, NWL determines the position of Timation II to ±20 m during the observation span. The positional accuracy outside of the observed data span remains near ±20 for extrapolations on the order of 12 to 24 hr. Beyond 1-day extrapolations, the error may grow rapidly.

For operational purposes the satellite ephemeris would require updating on a frequent basis. However, for the purpose of analyzing the Timation system performance, the analysis is routinely done using the satellite trajectory during the observed data span. This choice minimizes the contribution of satellite positional error to the overall system error.

COORDINATE SYSTEM

The satellite trajectory is given with respect to an earth-fixed coordinate system. Figure 4 depicts the coordinates of the satellite and the observer with respect to this frame. The z axis is the earth's polar axis, the x axis is along the Greenwich meridian in the equatorial plane, and the y axis completes the right-handed set. The satellite's position $\mathbf{r}$ and velocity $\mathbf{v}$ with respect to this set is given by

$$\mathbf{r} = (x, y, z)$$

and

$$\mathbf{v} = (v_x, v_y, v_z) \ .$$

The observer's coordinates with respect to this set is denoted by $\mathbf{r_s} = (x_s, y_s, z_s)$. The slant range between the observer and the satellite is given by

$$R = |\mathbf{r} - \mathbf{r_s}| \ . \tag{1}$$

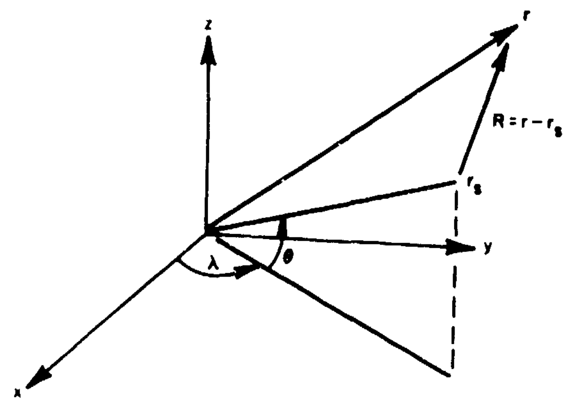

Fig. 4 - Earth-fixed coordinate system used to locate the satellite's position r and the observer r_s.

The rate of change of R is denoted as $\dot{R}$, and a unit vector along the slant range is given by

$$\hat{R} = \frac{(\mathbf{r} - \mathbf{r_s})}{|\mathbf{r} - \mathbf{r_s}|} . \tag{2}$$

The satellite ephemeris time base will be denoted by t_s and the observer's time base is denoted by t_g with the two bases related by

$$t_s \doteq t_g + \Delta t . \tag{3}$$

The quantity Δt should not be confused with the transmission time of the RF signal from the satellite to the observer. The time transmission delay is given by $-R/c$, where c is the speed of light in vacuo. A correction for the time transmission delay may be applied to either the measured data or the ephemeris. An explicit term for this will not be shown in the equations developed in the text, but the reader should keep in mind that this correction should be accounted for.

RANGE NAVIGATION EQUATIONS

The basic range navigation equations reflect the three quantities (a) time difference Δt between the observer's clock and the satellite's clock, (b) the observer's latitude, and (c) the observer's longitude. The frequency difference between the satellite and observer frequency standards is measured using values of Δt obtained from successive satellite passes taken from different positions. Denote the measured range by R_0 (the system measures time difference, which is readily converted to range by multiplication by c, the velocity of light) and the computed range (using an assumed position r_s for the observer) by R_c. Then the observed minus the computed time difference for each point measured during the satellite pass, denoted by $(O - C)_i$, is given by

$$(O - C)_i = (R_0)_i - (K + (R_c)_i) . \tag{4}$$

In Eq. (4), $K = c(\Delta t)$ where $\Delta t = t_s - t_g$ is the actual time difference between the satellite's and the observer's clocks. Now, a least-squares fit is preformed which adjusts these three quantities in such a way that the sum of squares of the range residuals are minimized.

The explicit function to be minimized, denoted by $\phi(K, \lambda, \theta)$, is given

$$\phi(K, \lambda, \theta) = \sum_{i=1}^{N} (O - C)_i^2 = \sum_{i=1}^{N} \left\{(R_0)_i - \left[(R_c)_i + K\right]\right\}^2 . \tag{5}$$

In Eq. (5), N is the number of range observations made during a satellite pass, and the computed function is $C_i = K + (R_c)_i$.

The technique used to solve Eq. (5) for a minimum is the least-square procedure (5) which involves the linearization of the computed function about assumed values of K, λ, and λ. In matrix form, the least-square solution is given by

$$B\Delta P = E \tag{6}$$

where

$$E = A^T W\,[O - C] , \quad \text{and} \quad B = A^T W A .$$

The 3×1 column vector ΔP has components $\Delta K, \Delta\lambda, \Delta\theta$ which are the corrections to the assumed values of K, λ, θ, the $N\times1$ column vector $O - C$ has the range residuals $(R_0)_i - [(R_c)_i + K]$ as components, A is the $N\times3$ matrix of partials of the computed function C_i (with respect to K, λ, θ) evaluated for each observation, W is the $N\times N$ weight matrix, and A^T is the A matrix transposed.

The required partial derivatives for the matrix A are $\partial C_i/\partial K$, $\partial C_i/\partial\lambda$, and $\partial C_i/\partial\theta$. The observer's position $\mathbf{r_s}$ is related to the latitude and longitude by the spherical-to-rectangular coordinate equations given by

$$\begin{aligned} x_s &= R_s \cos\theta \cos\lambda \\ y_s &= R_s \cos\theta \sin\lambda \\ z_s &= R_s \sin\theta . \end{aligned} \tag{7}$$

where θ is the geocentric latitude and λ is the longitude measured East from the Greenwich meridian.

The required partials are given by

$$\begin{aligned} \frac{\partial C_i}{\partial K} &= 1 \\ \frac{\partial C_i}{\partial\lambda} &= \frac{\partial(R_c)_i}{\partial x_s}\frac{\partial x_s}{\partial\lambda} + \frac{\partial(R_c)_i}{\partial y_s}\frac{\partial y_s}{\partial\lambda} = -\hat{\mathbf{R}} \cdot \frac{\partial \mathbf{r_s}}{\partial\lambda} \\ \frac{\partial C_i}{\partial\theta} &= \frac{\partial(R_c)_i}{\partial x_s}\frac{\partial x_s}{\partial\theta} + \frac{\partial(R_c)_i}{\partial y_s}\frac{\partial y_s}{\partial\theta} + \frac{\partial(R_c)_i}{\partial z_s}\frac{\partial z_s}{\partial\theta} = -\hat{\mathbf{R}} \cdot \frac{\partial \mathbf{r_s}}{\partial\theta} . \end{aligned} \tag{8}$$

The explicit derivatives required to form $\partial c_i/\partial\lambda$ and $\partial c_i/\partial\theta$ are given by

$$\frac{\partial R}{\partial \mathbf{r}_s} = \begin{bmatrix} \dfrac{\partial R}{\partial x_s} \\[2ex] \dfrac{\partial R}{\partial y_s} \\[2ex] \dfrac{\partial R}{\partial z_s} \end{bmatrix} = \begin{bmatrix} -\dfrac{(x - x_s)}{R} \\[2ex] -\dfrac{(y - y_s)}{R} \\[2ex] -\dfrac{(z - z_s)}{R} \end{bmatrix} = -\hat{\mathbf{R}} \ . \tag{9}$$

$$\frac{\partial \mathbf{r}_s}{\partial \lambda} = \begin{bmatrix} -R_s \cos\theta \sin\lambda \\ R_s \cos\theta \cos\lambda \\ 0 \end{bmatrix} , \tag{10}$$

and

$$\frac{\partial \mathbf{r}_s}{\partial \theta} = \begin{bmatrix} -R_s \sin\theta \cos\lambda \\ -R_s \sin\theta \sin\lambda \\ R_s \cos\theta \end{bmatrix} . \tag{11}$$

The complete differential correction matrix formulation is given by

$$\begin{bmatrix} N & \sum_{i=1}^{N} \dfrac{\partial (R_c)_i}{\partial \lambda} & \sum_{i=1}^{N} \dfrac{\partial (R_c)_i}{\partial \theta} \\[3ex] \sum_{i=1}^{N} \dfrac{\partial (R_c)_i}{\partial \lambda} & \sum_{i=1}^{N} \left(\dfrac{\partial (R_c)_i}{\partial \lambda}\right)^2 & \sum_{i=1}^{N} \left(\dfrac{\partial (R_c)_i}{\partial \lambda}\right)\left(\dfrac{\partial (R_c)_i}{\partial \theta}\right) \\[3ex] \sum_{i=1}^{N} \dfrac{\partial (R_c)_i}{\partial \theta} & \sum_{i=1}^{N} \left(\dfrac{\partial (R_c)_i}{\partial \lambda}\right)\left(\dfrac{\partial (R_c)_i}{\partial \theta}\right) & \sum_{i=1}^{N} \left(\dfrac{\partial (R_c)_i}{\partial \theta}\right)^2 \end{bmatrix} \begin{bmatrix} \Delta K \\[3ex] \Delta\lambda \\[3ex] \Delta\theta \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{N} (1)(O - C)_i \\[3ex] \sum_{i=1}^{N} \left(\dfrac{\partial (R_c)_i}{\partial \lambda}\right)(O - C)_i \\[3ex] \sum_{i=1}^{N} \left(\dfrac{\partial (R_c)_i}{\partial \theta}\right)(O - C)_i \end{bmatrix} \tag{12}$$

where the weight matrix W has been set to a diagonal matrix with weights equal to 1.

Another equivalent formulation in which the latitude and longitude are obtained by correcting the set x_s, y_s, z_s has been found useful. This formulation adds an equation of constraint, with R_s held constant for the observer, given by

$$G(x_s, y_s, z_s) = 0 = R_s^2 - (x_s^2 + y_s^2 + z_s^2) \ . \tag{13}$$

The complete matrix formulation, with the quantity Δg appearing as a result of the constraint procedure, is given by

$$
\begin{bmatrix}
N & \sum_{i=1}^{N} \frac{\partial (R_c)_i}{\partial x_s} & \sum_{i=1}^{N} \frac{\partial (R_c)_i}{\partial y_s} & \sum_{i=1}^{N} \frac{\partial (R_c)_i}{\partial z_s} & 0 \\
\sum_{i=1}^{N} \frac{\partial (R_c)_i}{\partial x_s} & \sum_{i=1}^{N} \left(\frac{\partial (R_c)_i}{\partial x_s}\right)^2 & \sum_{i=1}^{N} \left(\frac{\partial (R_c)_i}{\partial x_s}\right)\left(\frac{\partial (R_c)_i}{\partial y_s}\right) & \sum_{i=1}^{N} \left(\frac{\partial (R_c)_i}{\partial x_s}\right)\left(\frac{\partial (R_c)_i}{\partial z_s}\right) & 2x_s \\
\sum_{i=1}^{N} \frac{\partial (R_c)_i}{\partial y_s} & \sum_{i=1}^{N} \left(\frac{\partial (R_c)_i}{\partial x_s}\right)\left(\frac{\partial (R_c)_i}{\partial y_s}\right) & \sum_{i=1}^{N} \left(\frac{\partial (R_c)_i}{\partial y_s}\right)^2 & \sum_{i=1}^{N} \left(\frac{\partial (R_c)_i}{\partial y_s}\right)\left(\frac{\partial (R_c)_i}{\partial z_s}\right) & 2y_s \\
\sum_{i=1}^{N} \frac{\partial (R_c)_i}{\partial z_s} & \sum_{i=1}^{N} \left(\frac{\partial (k_c)_i}{\partial x_s}\right)\left(\frac{\partial (R_c)_i}{\partial z_s}\right) & \sum_{i=1}^{N} \left(\frac{\partial (R_c)_i}{\partial y_s}\right)\left(\frac{\partial (R_c)_i}{\partial z_s}\right) & \sum_{i=1}^{N} \left(\frac{\partial (R_c)_i}{\partial z_s}\right)^2 & 2z_s \\
0 & 2x_s & 2y_s & 2z_s & 0
\end{bmatrix}
\begin{bmatrix}
\Delta K \\ \Delta x_s \\ \Delta y_s \\ \Delta z_s \\ \Delta B
\end{bmatrix}
=
\begin{bmatrix}
\sum_{i=1}^{N} (1)(O-C)_i \\
\sum_{i=1}^{N} \frac{\partial (R_c)_i}{\partial x_s} (O-C)_i \\
\sum_{i=1}^{N} \frac{\partial (R_c)_i}{\partial y_s} (O-C)_i \\
\sum_{i=1}^{N} \frac{\partial (R_c)_i}{\partial z_s} (O-C)_i \\
0
\end{bmatrix}
. \quad (14)
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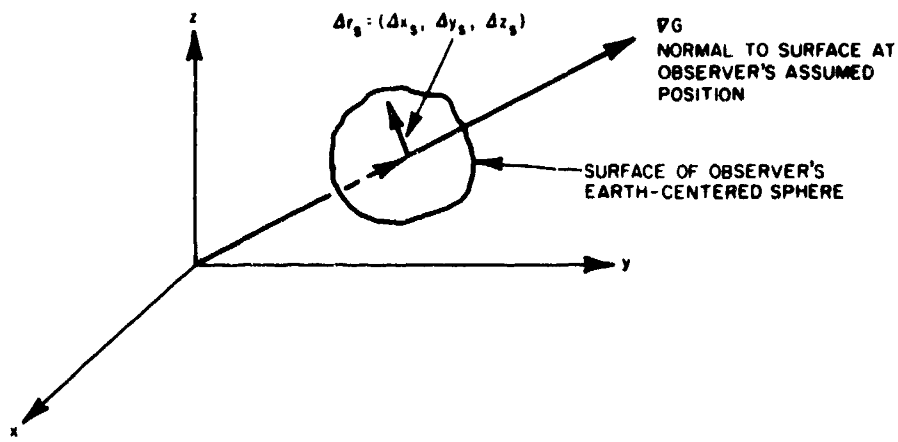

Fig. 5 - In one formulation of the range navigation equations, a constraint G is defined (see Eq. 13). This figure shows that the gradient of G is a vector perpendicular to the surface of the earth.

Figure 5 illustrates that the equation of condition is equivalent to saying that the dot product of $\mathbf{r}_s$ with a vector ∇G (gradient of G) normal to the surface at the assumed position of the observer equals zero.

A third useful formulation is obtained by constraining the solution to the plane determined by the velocity vector at the time of closest approach (denoted by TCA) of the satellite and the radius vector from the observer to the satellite at TCA.

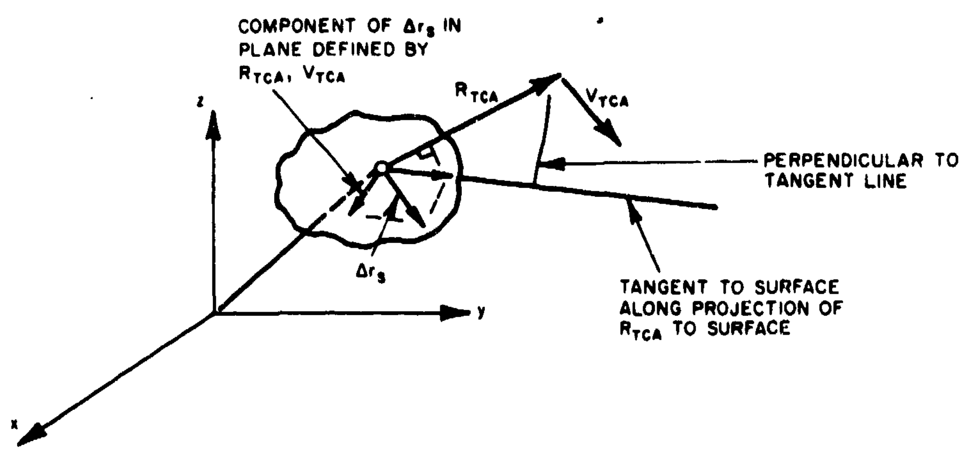

Fig. 6 - Arrangement of the pertinent range navigation vectors when the range solution is confined to the plane defined by $\mathbf{R}_{TCA}$ and $\mathbf{V}_{TCA}$.

Figure 6 shows that this procedure is equivalent to decomposing $\Delta\theta$, $\Delta\lambda$ into two components, one in the observer's horizontal plane, and the other a projection onto the radius vector at the TCA.

A vector perpendicular to the plane defined by $\mathbf{V}_{TCA}$ and $(\mathbf{r} - \mathbf{r}_s)_{TCA}$ is given by $\mathbf{T} = (\mathbf{r} - \mathbf{r}_s)_{TCA} \times \mathbf{V}_{TCA}$. This vector is constant, using the assumed value of $\mathbf{r}_s$; for an arbitrary $\mathbf{r}_s$ the constraint is expressed by

$$H = \mathbf{r}_s \cdot \mathbf{T} . \tag{15}$$

The partials $\partial H/\partial x_s$, $\partial H/\partial y_s$, and $\partial H/\partial z_s$ are required in the least-square formulation and are given by

$$\frac{\partial H}{\partial \mathbf{r}_s} = \begin{bmatrix} \dfrac{\partial H}{\partial x_s} \\[2ex] \dfrac{\partial H}{\partial y_s} \\[2ex] \dfrac{\partial H}{\partial z_s} \end{bmatrix} = \begin{bmatrix} (y - y_s)_{TCA} Vz_{TCA} - (z - z_s)_{TCA} Vy_{TCA} \\[2ex] -(x - x_s)_{TCA} Vz_{TCA} + (z - z_s)_{TCA} Vx_{TCA} \\[2ex] (x - x_s)_{TCA} Vy_{TCA} - (y - y_s)_{TCA} Vx_{TCA} \end{bmatrix} . \tag{16}$$

The matrix shown in Eq. (17) below gives the solution which will be referred to as the "along track" solution. Using ∇H obtained from Eq. (16), the last equation in the matrix of Eq. (17) expresses this constraint:

$$\begin{bmatrix} N & \sum_{i=1}^{N} \dfrac{\partial (R_c)_i}{\partial x_s} & \sum_{i=1}^{N} \dfrac{\partial (R_c)_i}{\partial y_s} & \sum_{i=1}^{N} \dfrac{\partial (R_c)_i}{\partial z_s} & 0 \\[2ex] \sum_{i=1}^{N} \dfrac{\partial (R_c)_i}{\partial x_s} & \sum_{i=1}^{N} \left(\dfrac{\partial (R_c)_i}{\partial x_s}\right)^2 & \sum_{i=1}^{N} \left(\dfrac{\partial (R_c)_i}{\partial x_s}\right)\left(\dfrac{\partial (R_c)_i}{\partial y_s}\right) & \sum_{i=1}^{N} \left(\dfrac{\partial (R_c)_i}{\partial x_s}\right)\left(\dfrac{\partial (R_c)_i}{\partial z_s}\right) & \dfrac{\partial H}{\partial x_s} \\[2ex] \sum_{i=1}^{N} \dfrac{\partial (R_c)_i}{\partial y_s} & \sum_{i=1}^{N} \left(\dfrac{\partial (R_c)_i}{\partial x_s}\right)\left(\dfrac{\partial (R_c)_i}{\partial y_s}\right) & \sum_{i=1}^{N} \left(\dfrac{\partial (R_c)_i}{\partial y_s}\right)^2 & \sum_{i=1}^{N} \left(\dfrac{\partial (R_c)_i}{\partial y_s}\right)\left(\dfrac{\partial (R_c)_i}{\partial z_s}\right) & \dfrac{\partial H}{\partial y_s} \\[2ex] \sum_{i=1}^{N} \dfrac{\partial (R_c)_i}{\partial z_s} & \sum_{i=1}^{N} \left(\dfrac{\partial (R_c)_i}{\partial x_s}\right)\left(\dfrac{\partial (R_c)_i}{\partial z_s}\right) & \sum_{i=1}^{N} \left(\dfrac{\partial (R_c)_i}{\partial y_s}\right)\left(\dfrac{\partial (R_c)_i}{\partial z_s}\right) & \sum_{i=1}^{N} \left(\dfrac{\partial (R_c)_i}{\partial z_s}\right)^2 & \dfrac{\partial H}{\partial z_s} \\[2ex] 0 & \dfrac{\partial H}{\partial x_s} & \dfrac{\partial H}{\partial y_s} & \dfrac{\partial H}{\partial z_s} & 0 \end{bmatrix} \begin{bmatrix} \Delta K \\[2ex] \Delta x_s \\[2ex] \Delta y_s \\[2ex] \Delta z_s \\[2ex] \Delta g \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{N} (1)(O-C)_i \\[2ex] \sum_{i=1}^{N} \left(\dfrac{\partial (R_c)_i}{\partial x_s}\right)(O-C)_i \\[2ex] \sum_{i=1}^{N} \left(\dfrac{\partial (R_c)_i}{\partial y_s}\right)(O-C)_i \\[2ex] \sum_{i=1}^{N} \left(\dfrac{\partial (R_c)_i}{\partial z_s}\right)(O-C)_i \\[2ex] 0 \end{bmatrix} . \tag{17}$$

The corrections Δx_s, Δy_s, Δz_s may be decomposed into components along the track of the satellite (ΔRV) and along the vector $\mathbf{R}_{TCA}$ which is denoted by ΔRR. This decomposition is useful for studying the effects of several error sources on the navigation solution, some which will be discussed later. The final latitude and longitude may be obtained by correcting x_s, y_s, z_s and converting from rectangular to spherical coordinates, or by obtaining the two components in the horizontal plane and then applying the parallelogram law to obtain the (vector) correction to the observer's assumed position.

The magnitude of $\Delta\mathbf{RR}$ and $\Delta\mathbf{RV}$ is given by

$$|\Delta\mathbf{RV}| = \frac{\mathbf{V}_{TCA}}{|\mathbf{V}_{TCA}|} \cdot \Delta\mathbf{r}_s \tag{18}$$

and

$$|\Delta\mathbf{RR}| = \hat{\mathbf{R}}_{TCA} \cdot \Delta\mathbf{r}_s \ . \tag{19}$$

The three formulations listed above do not exhaust the possible ways to obtain a position fix from the range data. Notice that in Eq. (4) the κ is assumed constant during the pass (up to 16 min duration). The reason for this is that the Timation System is designed to utilize the high-precision (3, 6) quartz crystal frequency standards available today. It should be emphasized that it is the uncertainty in the frequency that is important, inasmuch as the frequency offset due to crystal aging can be computed as a known correction to the measured data.

It is not necessary that the observer's frequency be precisely known in order to get a fix using Timation II. A different computed function may be used where the observed data is the range difference between two successive minutes. Note that continuous measurement of the range between minutes is not required to obtain this data. The navigation equations needed to utilize data in this fashion are related to navigation equations obtained by integrating the expression for doppler, which is sometimes called the "integrated doppler" solution. The relationship between "range difference" and "integrated doppler" will be discussed after the doppler navigation equations are derived.

What will be shown is that, for data treated in this manner, the clock difference (which may be expressed in terms of range by multiplication by c, the speed of light) is eliminated from the observed range data, and the bias parameter is the product of the navigator's frequency difference by the difference in observation time. The navigator's frequency difference is related as a constant rate of change of range through

$$\dot{R}_g = -\left(\frac{c}{f_s}\right)(f_s - f_g) \ . \tag{20}$$

Let the measured ranges for each minute be denoted by $(R_1^m, t_1), (R_2^m, t_2) \ldots (R_N^m, t_N)$ where $(t_2 - t_1) = \Delta T, (t_3 - t_2) = \Delta T$, etc. Let $\dot{R}_g$ be the rate of change of range due to the difference $(f_s - f_g)$ between the satellite frequency standard and the navigator's frequency standard. Let R_{g1} represent the range difference due to the difference $\Delta t = t_s - t_g$ between the satellite clock and the navigator's clock at time t_1. Then the measured ranges R_i^m may be separated to isolate the parts due to Δt and $f_s - f_g$ by

$$\begin{aligned}
R_1^m &= R_1 + R_{g1} + \dot{R}_g (t_1 - t_1) \\
R_2^m &= R_2 + R_{g1} + \dot{R}_g (t_2 - t_1) \\
R_3^m &= R_3 + R_{g1} + \dot{R}_g (t_3 - t_1) \\
&\;\;\vdots \qquad \vdots \\
R_N^m &= R_N + R_{g1} + \dot{R}_g (t_N - t_1) \ .
\end{aligned} \tag{21}$$

The measured ranges may be differenced to obtain

$$\begin{aligned} R_2^m - R_1^m &= (R_2 - R_1) + \dot{R}_g \Delta T \\ R_3^m - R_2^m &= (R_3 - R_2) + \dot{R}_g \Delta T \\ R_4^m - R_3^m &= (R_4 - R_3) + \dot{R}_g \Delta T \\ &\vdots \\ R_N^m - R_{N-1}^m &= (R_N - R_{N-1}) + \dot{R}_g \Delta T\ . \end{aligned} \tag{22}$$

Note that by differencing, the term R_{g1} which represents the clock difference Δt (expressed in terms of range) has been eliminated. A new computed function may now be defined as $C = K + [R(t + \Delta T) - R(t)]$, where $K = \dot{R}_g \Delta T$. The navigation equations for the "range difference" will be discussed after development of the doppler navigation equations.

The previous development illustrates that the range data may be processed with the frequency difference $(f_s - f_g)$ known or unknown. The decision as to which model the navigator should use will be influenced by the stability of his clock. It should be emphasized that if a user navigates using $K = \dot{R}_g \Delta T$, the clock difference $\Delta t = t_s - t_g$ may be recovered after the user's position has been determined by evaluating $R_1^m = R_1 + R_{g1}$, with $\Delta t = R_{g1}/c$.

DOPPLER NAVIGATION EQUATIONS

The basic doppler navigation equations reflect the three quantities (a) frequency difference between the observer's frequency standard and the satellite frequency standard, (b) the observer's latitude, and (c) the observer's longitude. The fourth required quantity, Δt, the difference between the observer's clock and the satellite ephemeris, is assumed to be known. The least-squares technique will again be used to obtain a solution; however the function to be minimized is the sum of the squares of the frequency residuals.

The computed doppler shift, denoted by DOP_{ci}, is given by Eq. (23), where f_s denotes the satellite system frequency.

$$DOP_{ci} = -\frac{f_s}{c} \mathbf{V}_{ci} \cdot \mathbf{R}_{ci} = -\frac{f_s}{c} \dot{R}\ . \tag{23}$$

Equation (23) may be rewritten to show the explicit dependence of doppler shift upon the position of the observer:

$$DOP_{ci} = -\left(\frac{f_s}{c}\right) \frac{V_{xi}(x_i - x_s) + V_{yi}(y_i - y_s) + V_{zi}(z_i - z_s)}{\sqrt{(x_i - x_s)^2 + (y_i - y_s)^2 + (z_i - z_s)^2}} \tag{24}$$

Note that in Eq.(24) no term is present to reflect the observer's velocity with respect to the earth-fixed set. This could be accomplished by replacing $\mathbf{V}_{ci}$ by $\mathbf{V}_{ci} - \mathbf{V}_s(t)$ where $\mathbf{V}_s(t)$ is the observer's velocity (known a priori).

The computed function is given by

$$C_i = K + \mathrm{DOP}_{c\,i} . \tag{25}$$

where K now represents the frequency difference (K assumed constant).

The observed minus computed function is given by

$$(O - C)_i = \mathrm{DOP}_{o\,i} - (K + \mathrm{DOP}_{c\,i}) . \tag{26}$$

The partial derivatives required to obtain a solution are lengthy, hence only one formulation will be given. The required partials for this formulation are given by

$$\begin{aligned}
\frac{\partial \mathrm{DOP}_{c\,i}}{\partial x_s} &= \left(\frac{f_s}{c}\right)\frac{V_{x\,i}}{R_i} + \mathrm{DOP}_{c\,i}\,\frac{(x_i - x_s)}{R_i^2} \\
\frac{\partial \mathrm{DOP}_{c\,i}}{\partial y_s} &= \left(\frac{f_s}{c}\right)\frac{V_{y\,i}}{R_i} + \mathrm{DOP}_{c\,i}\,\frac{(y_i - y_s)}{R_i^2} \\
\frac{\partial \mathrm{DOP}_{c\,i}}{\partial z_s} &= \left(\frac{f_s}{c}\right)\frac{V_{z\,i}}{R_i} + \mathrm{DOP}_{c\,i}\,\frac{(z_i - z_s)}{R_i^2} .
\end{aligned} \tag{27}$$

The matrix equation shown below gives the least-squares solution, which is a constrained solution for the latitude and longitude as a function of x_s, y_s, z_s:

$$\begin{bmatrix}
N & \sum_{i=1}^{N}\frac{\partial \mathrm{DOP}_{c\,i}}{\partial x_s} & \sum_{i=1}^{N}\frac{\partial \mathrm{DOP}_{c\,i}}{\partial y_s} & \sum_{i=1}^{N}\frac{\partial \mathrm{DOP}_{c\,i}}{\partial z_s} & 0 \\
\sum_{i=1}^{N}\frac{\partial \mathrm{DOP}_{c\,i}}{\partial x_s} & \sum_{i=1}^{N}\left(\frac{\partial \mathrm{DOP}_{c\,i}}{\partial x_s}\right)^2 & \sum_{i=1}^{N}\left(\frac{\partial \mathrm{DOP}_{c\,i}}{\partial x_s}\right)\left(\frac{\partial \mathrm{DOP}_{c\,i}}{\partial y_s}\right) & \sum_{i=1}^{N}\left(\frac{\partial \mathrm{DOP}_{c\,i}}{\partial x_s}\right)\left(\frac{\partial \mathrm{DOP}_{c\,i}}{\partial z_s}\right) & 2x_s \\
\sum_{i=1}^{N}\frac{\partial \mathrm{DOP}_{c\,i}}{\partial y_s} & \sum_{i=1}^{N}\left(\frac{\partial \mathrm{DOP}_{c\,i}}{\partial x_s}\right)\left(\frac{\partial \mathrm{DOP}_{c\,i}}{\partial y_s}\right) & \sum_{i=1}^{N}\left(\frac{\partial \mathrm{DOP}_{c\,i}}{\partial y_s}\right)^2 & \sum_{i=1}^{N}\left(\frac{\partial \mathrm{DOP}_{c\,i}}{\partial y_s}\right)\left(\frac{\partial \mathrm{DOP}_{c\,i}}{\partial z_s}\right) & 2y_s \\
\sum_{i=1}^{N}\frac{\partial \mathrm{DOP}_{c\,i}}{\partial z_s} & \sum_{i=1}^{N}\left(\frac{\partial \mathrm{DOP}_{c\,i}}{\partial x_s}\right)\left(\frac{\partial \mathrm{DOP}_{c\,i}}{\partial z_s}\right) & \sum_{i=1}^{N}\left(\frac{\partial \mathrm{DOP}_{c\,i}}{\partial y_s}\right)\left(\frac{\partial \mathrm{DOP}_{c\,i}}{\partial z_s}\right) & \sum_{i=1}^{N}\left(\frac{\partial \mathrm{DOP}_{c\,i}}{\partial z_s}\right)^2 & 2z_s \\
0 & 2x_s & 2y_s & 2z_s & 0
\end{bmatrix}
\begin{bmatrix} \Delta K \\ \Delta x_s \\ \Delta y_s \\ \Delta z_s \\ \Delta g \end{bmatrix}
=
\begin{bmatrix}
\sum_{i=1}^{N}(1)(O - C)_i \\
\sum_{i=1}^{N}\left(\frac{\partial \mathrm{DOP}_{c\,i}}{\partial x_s}\right)(O - C)_i \\
\sum_{i=1}^{N}\left(\frac{\partial \mathrm{DOP}_{c\,i}}{\partial y_s}\right)(O - C)_i \\
\sum_{i=1}^{N}\left(\frac{\partial \mathrm{DOP}_{c\,i}}{\partial z_s}\right)(O - C)_i \\
0
\end{bmatrix} \tag{28}$$

Doppler solutions in terms of the bias parameter K, the along-track error ΔRV, and the slant range error ΔRR are obtained in addition to the solution for latitude and longitude.

The measured doppler shift DOP_0 is obtained from the 1 k (1000 cycles), 10 k, or 40 k period average data available with some of the Timation II equipment. To obtain the conversion, let t_0 be the midpoint of the measurement interval, N the number of cycles counted by the equipment, and T_N the total amount of time required to count N cycles of the offset doppler signal. The ground station time base t_g is used to obtain the count time T_N. Let f represent the frequency that is counted, which is given by

$$f = -\frac{f_s}{c}\dot{R}(t_0) + f_Q + (f_s - f_g) \tag{29}$$

where the first term is the doppler shift at t_0, f_Q is a constant offset in the ground station equipment, and $f_s - f_g$ represents the difference between the satellite and ground station frequency standards. The number of cycles N obtained by counting f over a time interval T_N centered about point t_0 is given by

$$N = \int_{t_0 - T_N/2}^{t_0 + T_N/2} f dt = -\frac{f_s}{c}[R(t_0 + T_N/2) - R(t_0 - T_N/2)] + [f_Q + (f_s - f_g)]T_N . \tag{30}$$

Since R may be expanded about the point t_0, Eq. (30) may be approximated by

$$N = \left(-\frac{f_s}{c}\dot{R} + (f_Q + K) + \dddot{R}\frac{(t_0)}{24} T_N^2\right) T_N . \tag{31}$$

The term $\dddot{R}(t_0) T_N^2/24$ is the measure of the error which is committed in obtaining the measured doppler shift DOP_0 given by

$$DOP_0 = \left(\frac{N}{T_N}\right) - f_Q \tag{32}$$

Figure 7 is a plot of the maximum error committed by using Eq. (32) to obtain the measured doppler. However, a correction has been applied in the results for this term so that, for the longest count used 40,000 cycles, the representation error will be considerably smaller than the measurement error.

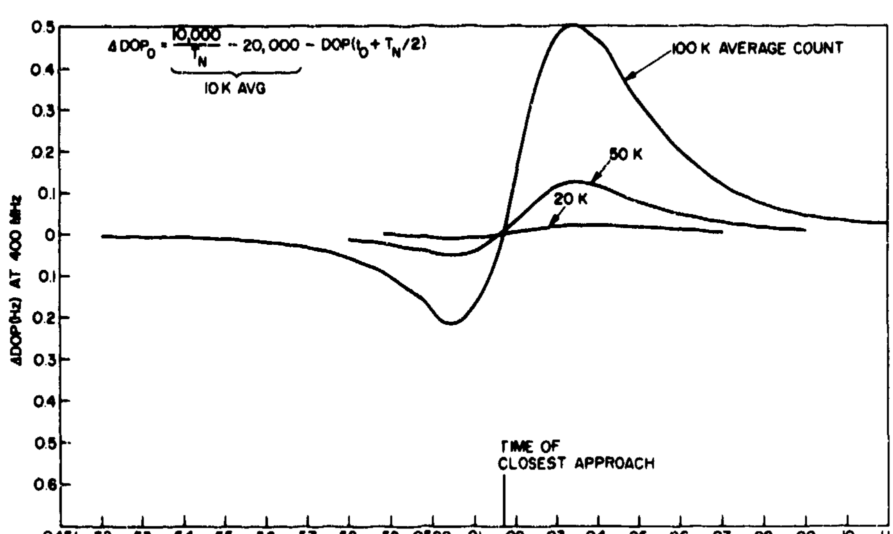

Fig. 7 - Maximum error committed by using the relation $DOP_0 = (N/T) - f_Q$ for the measured doppler.

A navigation may also be performed using Eq. (30), which is sometimes called the "integrated doppler" solution. Comparison of Eq. (30) with the range difference Eq. (21) shows the relationship between the "integrated doppler" and the range differences obtained by differencing the range data. The factor $-f_s/c$ is a constant and the extra term $f_Q T_N$ in Eq. (30) is constant whenever T_N is specified by the user. The factor f_Q is dependent on the counting equipment used, and the NRL ground station uses an offset of 20 kHz. The term $(f_s - f_g) T_N$ may now be related to the term $\dot{R}_g$ in the range difference solution by

$$(f_s - f_g) = -\left(\frac{f_s}{c}\right) \dot{R}_g \ . \tag{32}$$

It can now be seen that Eqs. (30) and (21) are related even though the measurements of "range difference" and "integrated doppler" are obtained by two independent measuring techniques. As stated previously, the quantity $\Delta t = t_s - t_g$ cannot be obtained by the "integrated doppler" technique, and therefore the clock correction cannot be obtained by this method.

The navigation equations for "integrated doppler" will be derived, with the "range difference" equations obtained by using Eq. (30), multiplying the first term by $-c/f_s$, and removing the term $f_Q T_N$ from the computed function.

The auxiliary function $D(t, T_N)$, given by

$$D(t, T_N) = -\frac{f_s}{c}\,[R(t_0 + T_N/2) - R(t_0 - T_N/2)] \ ,$$

is defined for convenience, and $K = (f_s - f_g) T_N$ will be the bias parameter.

Now the computed function is given by

$$C_i = K + D_i(t_i, T_N) + f_Q T_N \ . \tag{33}$$

The required partials are given by

$$\begin{aligned}
\frac{\partial C_i}{\partial K} &= 1 \\
\frac{\partial C_i}{\partial x_s} &= \frac{\partial D_i}{\partial x_s} = -\frac{f_s}{c}\left[\frac{\partial R}{\partial x_s}(t_i + T_N/2) - \frac{\partial R}{\partial x_s}(t_i - T_N/2)\right] \\
\frac{\partial C_i}{\partial y_s} &= \frac{\partial D_i}{\partial y_s} = -\frac{f_s}{c}\left[\frac{\partial R}{\partial y_s}(t_i + T_N/2) - \frac{\partial R}{\partial y_s}(t_i - T_N/2)\right] \\
\frac{\partial C_i}{\partial z_s} &= \frac{\partial D_i}{\partial z_s} = -\frac{f_s}{c}\left[\frac{\partial R}{\partial z_s}(t_i + T_N/2) - \frac{\partial R}{\partial z_s}(t_i - T_N/2)\right] \ .
\end{aligned} \tag{34}$$

The equations given will be using the constraint given in Eq. (35) below. However, the along-track solution, or the direct solution for latitude and longitude, may be obtained in a manner similar to the technique used for the corresponding range solutions.

The frequency difference $f_s - f_g$ may be determined if ΔT is a variable, that is, the product $(f_s - f_g)\Delta T$ is not constant. This may be done by setting $C_i = (f_s - f_g)\Delta T_i + D(t_i, T_i)$ and using the constant term as $f_s - f_g$. This may be done by replacing $\partial C_i/\partial k = 1$ by $\partial C_i/\partial(f_s - f_g) = \Delta T_i$ in the normal equations.

$$
\begin{bmatrix}
N & \sum_{i=1}^{N} \frac{\partial D_i}{\partial x_s} & \sum_{i=1}^{N} \frac{\partial D_i}{\partial y_s} & \sum_{i=1}^{N} \frac{\partial D_i}{\partial z_s} & 0 \\
\sum_{i=1}^{N} \frac{\partial D_i}{\partial x_s} & \sum_{i=1}^{N} \left(\frac{\partial D_i}{\partial x_s}\right)^2 & \sum_{i=1}^{N} \left(\frac{\partial D_i}{\partial x_s}\right)\left(\frac{\partial D_i}{\partial y_s}\right) & \sum_{i=1}^{N} \left(\frac{\partial D_i}{\partial x_s}\right)\left(\frac{\partial D_i}{\partial z_s}\right) & 2x_s \\
\sum_{i=1}^{N} \frac{\partial D_i}{\partial y_s} & \sum_{i=1}^{N} \left(\frac{\partial D_i}{\partial x_s}\right)\left(\frac{\partial D_i}{\partial y_s}\right) & \sum_{i=1}^{N} \left(\frac{\partial D_i}{\partial y_s}\right)^2 & \sum_{i=1}^{N} \left(\frac{\partial D_i}{\partial y_s}\right)\left(\frac{\partial D_i}{\partial z_s}\right) & 2y_s \\
\sum_{i=1}^{N} \frac{\partial D_i}{\partial z_s} & \sum_{i=1}^{N} \left(\frac{\partial D_i}{\partial x_s}\right)\left(\frac{\partial D_i}{\partial z_s}\right) & \sum_{i=1}^{N} \left(\frac{\partial D_i}{\partial y_s}\right)\left(\frac{\partial D_i}{\partial z_s}\right) & \sum_{i=1}^{N} \left(\frac{\partial D_i}{\partial z_s}\right)^2 & 2z_s \\
0 & 2x_s & 2y_s & 2z_s & 0
\end{bmatrix}
\begin{bmatrix}
\Delta K \\ \Delta x_s \\ \Delta y_s \\ \Delta z_s \\ \Delta g
\end{bmatrix}
=
\begin{bmatrix}
\sum_{i=1}^{N} (1)(O-C)_i \\
\sum_{i=1}^{N} \left(\frac{\partial D_i}{\partial x_s}\right)(O-C)_i \\
\sum_{i=1}^{N} \left(\frac{\partial D_i}{\partial y_s}\right)(O-C)_i \\
\sum_{i=1}^{N} \left(\frac{\partial D_i}{\partial z_s}\right)(O-C)_i \\
0
\end{bmatrix}
\quad (35)
$$

This concludes the development of the navigation equations for range, range difference, doppler, and integrated doppler measurements. A data weighting matrix may also be used according to the techniques given by Ref. (5); however this was not done in this development in order to simplify notation. Other iterative techniques exist, and it should be mentioned that graphical techniques (7) may be used to obtain solutions for either range or doppler position fixes.

RANGE/DOPPLER NAVIGATION EQUATIONS

The simultaneous measurement of both range and doppler offers the potential of an observer obtaining a two-dimensional fix (latitude and longitude) "instantaneously." In order to do this the range and doppler bias parameters must be known. This requirement indicates than an atomic frequency standard would be desirable for the user. The atomic frequency standards commercially available are capable of extremely stable phase and frequency operation. Hence a system user equipped with an atomic frequency standard could place a close tolerance on both the range and frequency bias parameters for a relatively long interval of time after some initial calibration. Note also the fact that some of the ultrahigh-precision quartz crystal oscillators may also be used, but for shorter intervals of time between calibrations, which may be accomplished by periodically taking more data than the two points required for an "instantaneous fix."

The three equations that are used to obtain a solution, using an assumed position, are given by

$$|\mathbf{r} - \mathbf{r_s}| = R \ ,$$

$$-\frac{f_s}{c}\,\mathbf{v} \cdot \frac{(\mathbf{r} - \mathbf{r_s})}{|\mathbf{r} - \mathbf{r_s}|} = \mathrm{DOP} \ ,$$

and

$$|\mathbf{r_s}| = R_s \ , \tag{36}$$

where the first two equations are repeats of Eqs. (1) and (23).

These three equations are rewritten slightly for convenience as

$$-\frac{f_s}{c}\left[v_x(x - x_s) + v_y(y - y_s) + v_z(z - z_s)\right] = R(\mathrm{DOP}) \ , \tag{37}$$

$$(x - x_s)^2 + (y - y_s)^2 + (z - z_s)^2 = R^2 \ , \tag{38}$$

and

$$x_s^2 + y_s^2 + z_s^2 = R_s^2 \ . \tag{39}$$

The observed quantities are denoted by R_0 and DOP_0. There is no need to use least squares to get a solution here because we have exactly three equations of condition in three unknowns (x_s, y_s, z_s). The differential correction equations are given by the matrix

$$\begin{bmatrix} \frac{f_s}{c} v_x & \frac{f_s}{c} v_y & \frac{f_s}{c} v_z \\ -(x - x_s) & -(y - y_s) & -(z - z_s) \\ x_s & y_s & z_s \end{bmatrix} \begin{bmatrix} \Delta x_s \\ \Delta y_s \\ \Delta z_s \end{bmatrix} = \begin{bmatrix} DOP_0 R_0 - \left(\frac{f_s}{c}\right) \mathbf{v} \cdot (\mathbf{r} - \mathbf{r}_s) \\ (R_0^2 - R^2)/2 \\ 0 \end{bmatrix} . \tag{40}$$

It should be noted that a polynomial solution in one of the three coordinates can be obtained by suitable manipulation. Undoubtedly, a trigonometric solution in terms of latitude or longitude could be obtained by expressing the range and doppler as functions of θ and λ. However, Eq. (40) is relatively easy to compute and converges well for any reasonable guess of the observer's position $\mathbf{r}_s$.

FIRST-ORDER RANGE IONOSPHERIC CORRECTIONS

The ranging information at 150 and 400 MHz may be combined to measure and remove first-order ionospheric refraction effects. In Fig. 8 denote the actual transmission times for the 150- and 400-MHz signals by T_{150} and T_{400}, respectively. Equations (41) and (42) give an expression for the total path length, where n is the index of refraction (along the optical path), ds is the element of arc length, and P_T and P_R denote the position of the satellite's transmitting antenna and the observer's receiving antenna, respectively:

$$T_{150} = \frac{1}{c} \int_{P_T}^{P_R} n'_{150} \, ds \tag{41}$$

$$T_{400} = \frac{1}{c} \int_{P_T}^{P_R} n'_{400} \, ds . \tag{42}$$

Although the path of integration is slightly different for each signal, each path is close to the geometric path along $\mathbf{R}$.

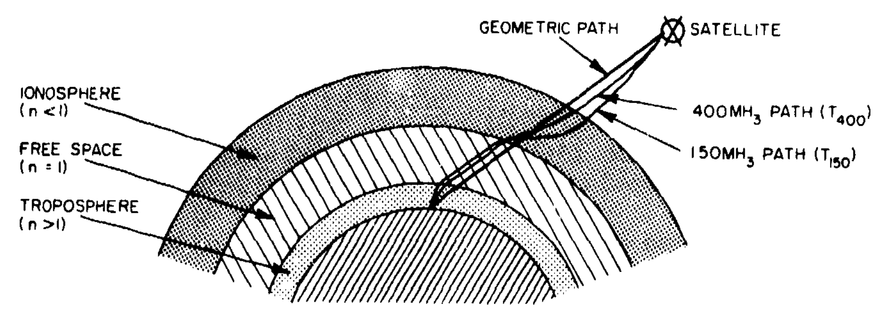

Fig. 8 - Typical ray path trajectory. The quantities T_{150} and T_{400} give the actual transmission times for the 150- and 400-MHz ranging signals, respectively.

The index of refraction will be split into two parts, with Δn_{trop} representing the error ($\Delta n_{trop} = n_{trop} - 1$) due to the troposphere, which is assumed to be independent of frequency. That is, $n'_{150} = n_{150} + \Delta n_{trop}$, and $n'_{400} = n_{400} + \Delta n_{trop}$.

The ionospheric refraction may be expanded in the form given by

$$n - 1 = \frac{\alpha}{f^2} + \frac{\beta}{f^3} \cdots \quad (43)$$

with α and β dependent upon time and position.

Let f_1 denote the 150-MHz frequency and f_2 denote the 400-MHz frequency. Also note that $8f_1 = 3f_2$. The actual ratio (3/8) is slightly different from an exact ratio, but the form of the correction may be easily changed to accommodate any ratio. Substituting Eqs. (41) and (42) into (43), and noting that for $n = 1$ we obtain the geometric path from P_T to P_R, we obtain

$$T_{150} = \int_{P_T}^{P_R} \Delta n_{trop} ds + \frac{R}{c} - \int_{P_T}^{P_R} \frac{\alpha ds}{f_1^2} + \int_{P_T}^{P_R} \frac{\beta ds}{f_1^3} \cdots \quad (44)$$

and

$$T_{400} = \int_{P_T}^{P_R} \Delta n_{trop} ds + \frac{R}{c} - \int_{P_T}^{P_R} \frac{\alpha ds}{f_2^2} + \int_{P_T}^{P_R} \frac{\beta ds}{f_2^3} \cdots \quad (45)$$

Now using the relation $8f_1 = 3f_2$, the first-order ionospheric term is eliminated, resulting in

$$\frac{R}{c} = T_{400} + \frac{9}{55}(T_{400} - T_{150}) - \int_{P_T}^{P_R} \Delta n_{trop} ds + \frac{320}{165} \int_{P_T}^{P_R} \frac{\beta ds}{f_1^3} \cdots \quad (46)$$

Let ΔR_{150} and ΔR_{400} denote the first-order corrections to the 150- and 400-MHz ranges, respectively. Then

$$\Delta R_{400} = \frac{9c}{55}(T_{400} - T_{150}) = \frac{3^2 c}{8^2 - 3^2}(T_{400} - T_{150}) \quad (47)$$

$$\Delta R_{150} = \frac{64c}{55}(T_{400} - T_{150}) = \frac{8^2 c}{8^2 - 3^2}(T_{400} - T_{150}) . \quad (48)$$

The tropospheric term in Eq. (46) can be calculated using measurements, or a standard atmosphere can be used to obtain an approximate value.

The form of Eqs. (47) and (48) is such that it appears to be easier to make the correction by computer rather than by additional circuitry.

SINGLE-PASS GEOMETRY

An important consideration to a system user is the geometry presented by a system configuration and the influence of system geometry and overall system noise to navigational accuracy. The range and doppler navigation equations derived in this report may be used for data gathered in one pass of the satellite, which may be as much as 16 min in duration. Reference 7 gives other possible satellite configurations for Timation.

A weighted least-squares analysis may be made with the data which includes such factors as the uncertainty in the estimate of the assumed position and the correlations present in the observed data with appropriate weighting factors. Such a weighted least-squares formulation may be obtained using techniques derived in Ref. 8. Equation (49) below gives one such formulation where $\bar{\Gamma}_x$ is the a priori state covariance matrix, Γ_x is the "post-flight" covariance matrix, A is the matrix of observational partials as defined in this report, W is a weight matrix, and Γ_z is the data covariance matrix:

$$\Gamma_x = (A^T WA + \bar{\Gamma}_x^{-1})^{-1}(A^T W \Gamma_z WA + \bar{\Gamma}_x^{-1})(A^T WA + \bar{\Gamma}_x)^{-1} \quad . \tag{49}$$

The geometry may be singled out for study by assuming that (a) $\bar{\Gamma}_x^{-1} = 0$, (b) Γ_z is uncorrelated with $\Gamma_z^{-1} = W$, and (c) $W = (1/\sigma^2) I$. With these assumptions, Eq. (48) simplifies to

$$\Gamma_x = \sigma^2 (A^T A)^{-1} = \sigma^2 B^{-1} \quad . \tag{50}$$

Now Eq. (50) may be evaluated for a selected single-pass geometry, using the appropriate equations for range or doppler. Figure 9 indicates that for the single-pass geometry the important factors are (a) maximum elevation angle, (b) symmetrical data, and (c) amount of data used in the solution. It should be emphasized that this analysis is only for the single-pass geometry with a 500-naut-mi near-circular orbit.

Figure 9 also shows the relative effect of satellite geometry upon navigation accuracy. Bold lines indicate passes with maximum elevation angles of 15° and 75° with 12 to 16 min of data available. Any pass with maximum elevation between 15° and 75° and 7 to 15 min of data has relative weight near 1.

The remaining lines indicate only partial data collection, varying from minimum data (3 points) to half of the pass. These relative weights may be used to estimate a fix accuracy: (a) data, and (b) multiplying by the total system error, which includes the measurement noise, satellite position uncertainty, etc.

The relative navigation error is given as a function of the data span (or time) in Fig. 10, where it is assumed that the maximum elevation angle is between 15 and 75° with the data taken symmetrically with respect to the maximum elevation point. For example, the figure shows a factor of 20 in the error distribution of a collection of navigations obtained using a 15-min data span versus a collection of navigations obtained using a 3-min data span.

It should be noted that the discussion has been limited to the determination of three navigational parameters (latitude, longitude, and a bias parameter) from a single-pass configuration which allows up to 16 min of data collection.

CONCLUSIONS

It is concluded that, using the techniques outlined in this report, (a) a navigator can determine his position and synchronize his clock by using the passive ranging technique, (b) the measured range data may be used to obtain a fix regardless of whether the user's frequency is known or unknown, and (c) the time difference may be obtained in either case. The doppler data can be used to obtain a fix provided that clock synchronization

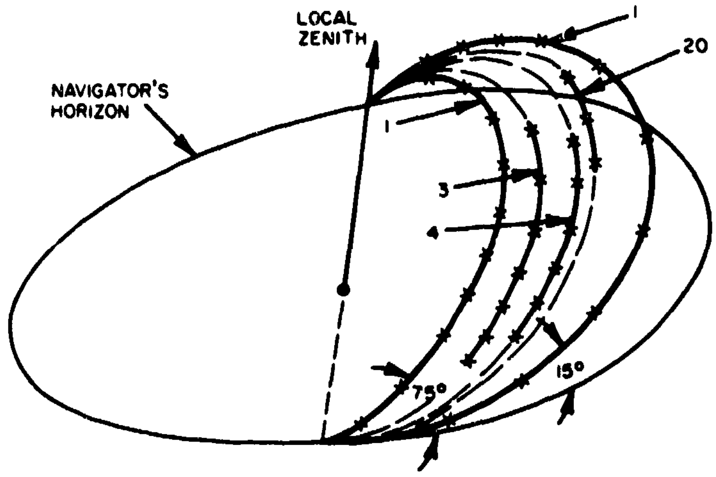

Fig. 9 - Single-pass geometry associated with a near-circular 500-naut-mi orbit. The x's on each trajectory represent times at which navigational data might be obtained using the satellite. The numbers are relative GDOP (Geometric Dilution of Precision) factors which indicate the effect of partial data collection during a satellite pass.

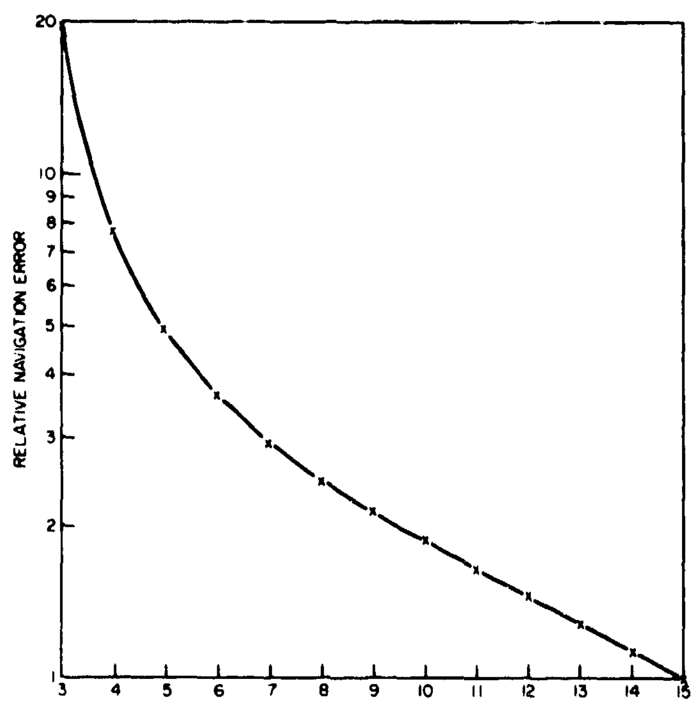

Fig. 10 - Relative navigation error as a function of the time during which data is obtained (called the data span). For example, the relative error is 20 times as great for a 3-min data span as compared to a 15-min data span.

is obtained from an independent source, and that clock synchronization cannot be obtained from integrated doppler data. The range and doppler data may be used to obtain an instantaneous fix, assuming the use of suitable techniques. It is further shown that for the 500-naut-mi single-pass configuration the maximum elevation angle, the amount of data, and the symmetrical character of the data are important factors in navigation fix accuracy.

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